IKKI O'LCHOVLI DISKRET DINAMIK SISTEMALARNI O'QITISHDA GRAFIK TALQIN

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Nishonov Samat Nishonovich
Termiz davlat universiteti katta o'qituvchisi

Seytov Shavkat
O'zbekiston Milliy universiteti tayanch doktoranti

Annotatsiya: Ushbu maqolada ikki o'lchovli diskret dinamik sistemalarni o'rganishda grafiklardan foydalanish o'rganilgan. Ikki o'lchovli kvadratik akslantirish uchun qo'zg'almas nuqtalar, grafik tahlil usuli, Julia to'plami, Mandelbrot to'plami xossalari sonli va grafik usulda keltirilgan.

Kalit so'zlar: Qo'zgalmas nuqta, davriy nuqta, bifurkatsiya, Julia to'plami, Mandelbrot to'plami.

ДВУХМЕРНЫЕ ДИСКРЕТНЫЕ ДИНАМИЧЕСКИЕ СИСТЕМЫ ГРАФИЧЕСКАЯ ИНТЕРПРЕТАЦИЯ В ОБУЧЕНИИ

Нишонов Самат Нишович
Старший преподаватель Термезского государственного университета

Шавкат Сейидов
Базовый докторант Национального университета Узбекистана

Аннотация: В данной статье изучалось использование графиков при изучении двумерных дискретных динамических систем. Для двумерного квадратичного отражения свойства неподвижных точек, графического метода анализа, множества Юлиана, множества Мандельброта представлены численным и графическим методами.

Ключевые слова: неподвижная точка, периодическая точка, бифуркация, коллекция Джулии, коллекция Мандельброта.

GRAPHICAL INTERPRETATION FOR THE STUDY OF TWO DIMENSIONAL DISCRETE DYNAMICAL SYSTEMS

Nishonov Samat Nishonovich
Senior lecturer at Termez State University

Seytov Shavkat
Base doctoral student of the National University of Uzbekistan

Annotation: this article examines the use of graphs in the study of two-dimensional discrete dynamic systems. The properties of fixed points for two-dimensional quadratic reflection, the method of graphical analysis, the set of Julia, the set of Mandelbrot are presented in the numerical and graphical way.

Keywords: fixed point, davriy point, bifurcation, Julia collection,Mandelbrot collection.

Suppose we have the two-dimensional mapping

$$Q_{c_1c_2}:\begin{cases} x' = f(y, c_1) \\ y' = f(x, c_2) \end{cases}$$

From [3] we know that graphical analysis of (x_0, y_0) for $Q_{c_1c_2} = \begin{cases} x' = y^2 + c_1 \\ y' = x^2 + c_2 \end{cases}$

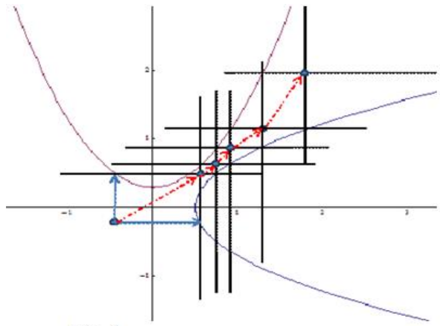


Fig. 1.

It is tame that $Q_{c_1c_2}$ quadratic mapping maps the vertical interval to the horizontal interval and inversely. Interval may be contract or extend. This proves the following statement.

Lemma. There are may be only one fixed point on one horizontal and one vertical line for (1).

Let the graph of $x = f(y, c_1)$ tangents with the graph of $y = f(x, c_2)$ externally and denote tangent point by (a, b) . Then this point is fixed point for $Q_{c_1c_2}$.

Definition 1. The filled-in Julia set for quadratic operator Q_{ab} is

$$K_{Q_{ab}} = \{X \in R^2 \mid the\ orbit\ \{Q_{ab}^{(n)}(X)\}_{n=0}^{\infty}\ is\ bounded\}$$

Then its boundary coincides with the **Julia set**: $J_{Q_{ab}} = \partial K_{Q_{ab}}$

Definition 2. The basin of attraction point (x_p, y_p) of quadratic operator Q_{ab} is the set of points (x, y) such that $\left|Q_{ab}^n(x, y) - Q_{ab}^n(x_p, y_p)\right| \to 0$ as $n \to \infty$.

Definition 3. The Mandelbrot set $M_{Q_{ab}}$ for quadratic mapping Q_{ab} is the set of points in (a, b) parameter plane, which the orbits of the all critical points are bounded.

$Q_{c_1c_2}$ quadratic mapping maps the rectangles to the rectangles, length and wide may be contract or extend.

Theorem 1. Graphical analysis shows that the points does not belong to rectangle with vertices $(\pm a, \pm b)$ are tend to infinite. The points belong to rectangle with vertices $(\pm a, \pm b)$ are tend to (a, b). It means the set all points belong to rectangle with vertices $(\pm a, \pm b)$ is the filled Julia set for $Q_{c_1c_2}$.

Theorem 2. Orbits of every points by $Q_{c_1c_2} = \begin{cases} x' = y^2 + c_1 \\ y' = x^2 + c_2 \end{cases}$ in the Julia set, does not go out from the box $c_1 \le x \le \sqrt{c_1 - c_2}$, $c_2 \le y \le c_2^2 + c_2$.

We denote the greater fixed point of $Q_{c_1c_2}$ by (a, b) then other fixed points.

$$\begin{cases} a = b^2 + c_1 \\ b = a^2 + c_2 \end{cases} \Rightarrow \begin{cases} c_1 = a - b^2 \\ c_2 = b - a^2 \end{cases} \Rightarrow Q_{ab} = \begin{cases} x' = y^2 + a - b^2 \\ y' = x^2 + b - a^2. \end{cases} \qquad (2)$$

When the bottoms of parabolas belong to rectangle with vertices $(\pm a, \pm b)$ the Julia set is totally connected set and this rectangle. If one of parabola's bottom does not belong to $(\pm a, \pm b)$ rectangle the Julia set changes from totally connected set $b - a^2 \geq -b \;\; and \;\; a - b^2 \geq -a \Rightarrow a^2 \leq 2b, b^2 \leq 2a$. Let the bottom of $x = y^2 + a - b^2$ is out from $(\pm a, \pm b)$ rectangle $a^2 > 2b$. But the bottom of $y = x^2 + b - a^2$ belongs to $(\pm a, \pm b)$ rectangle in this case $b^2 \leq 2a$ Fig. 2.

It is easy to find the coordinates of C, C', D, D'. $C\left(-a, \sqrt{b^2 - 2a}\right), C'\left(-a, -\sqrt{b^2 - 2a}\right), D\left(a, \sqrt{b^2 - 2a}\right), D'\left(a, -\sqrt{b^2 - 2a}\right)$. If the bottom of $y = x^2 + b - a^2$ belongs to $ABCD$ then the Julia set is consist of $ABCD \; and \; A'B'C'D'$ disconnected rectangles. $A'B'C'D'$ is the pre-image of $ABCD$. The orbit of every point in $ABCD$ leaves in $ABCD$.

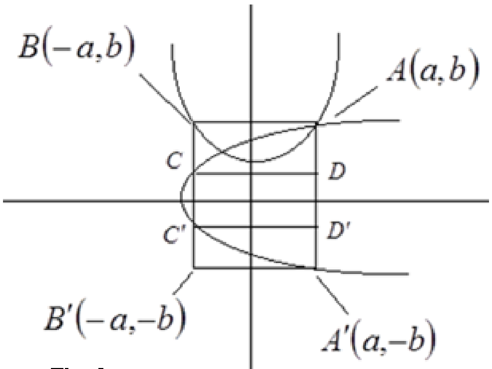


Fig. 2.

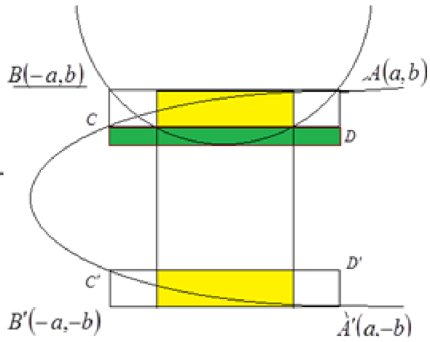


Fig. 3.

The orbit of every point in $A'B'C'D'$ jumps to in $ABCD$ at the first iteration. The set of points below CD and above $C'D'$ go out from $ABCD$ at one iteration and tend to infinity. The set of such all points which go out from $ABCD$ at one iteration we denote by $M_1, Q_{ab}(M_1) \not\subset ABCD$.

Let the bottom of $y = x^2 + b - a^2$ is below CD. Fig. 3. It is tame by graphical analysis all point belong to yellow rectangles jump to green rectangle at one iteration and at second iteration go out from $ABCD$. The vertical ends of yellow rectangles do not go out from $ABCD$. The set of such all points which go out from $ABCD$ at two iterations we denote by M_2, $Q_{ab}(M_2) \subset M_1$. We continue such fashion the set of such all points which go out from $ABCD$ at tree iterations we denote by $M_3, Q_{ab}(M_3) \subset M_2$. M_n is the set of such all points which go out from $ABCD$ at n iterations $Q_{ab}(M_n) \subset M_{n-1}$. Are there any points left after we throw out all of these rectangles from $ABCD$?

The ends of rectangles and the fixed points, eventually fixed points, periodic points, eventually periodic points left in $ABCD$? The set of all points left in $ABCD$ forever is Cantor type set we denote in by Λ

$$\Lambda = \{|x| \leq a, |y| \leq b\} \setminus \sum_{i=1}^{\infty} M_i.$$

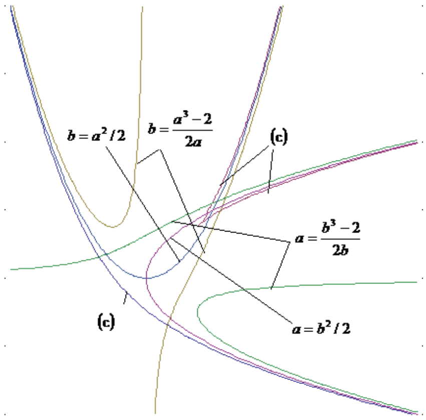
The $b = a^2/2$, $a = b^2/2$, $b^4 - 2ab^2 + 2b = 0$, $a^4 - 2a^2 b + 2a = 0$ curves mean some boundaries of Mandelbrot set. We know from [1]

$4ab + 12(a+b) - 2\sqrt{(2a+2b+3)^3} + 9 = 0$ and $4ab + 12(a+b) + 2\sqrt{(2a+2b+3)^3} + 9 = 0$ curves some boundaries of Mandelbrot set.

Theorem 3. The closed set bounded with curves which graphics of following functions on $R^2 = (a,b)$ parameter space

(a) $b = a^2/2$ and $a = b^2/2$. (b) $b = \frac{a^3-2}{2a}$ and $a = \frac{b^3-2}{2b}$ (c) $4ab = 1$

coincides with the boundaries of the set of Mandelbrot for quadratic mapping Q_{ab} .



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