METHODS FOR SOLVING FUNCTIONAL EQUATIONS.

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Abstract. The article describes functional equations and how functional equations are solved. The article shows some typical tricks for solving functional equations. It also explains the method of substitution which consists in the fact that variables are replaced by some new functions.

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Introduction

You are familiar with functional equations, dear students, although you may not know that they are called that. So, it is the functional equations $f(x)=f(-x)$, $f(-x)=-f(x)$, $f(x+T)=f(x)$ that define such properties of functions as parity, oddness, periodicity.

A functional equation is an equation that contain one or more unknown functions (with specified domains and values).

To solve a functional equation means to find all functions that satisfy it identically. Functional equations arise in a wide variety of areas of mathematics, usually in those cases when it is required to describe all functions that have given properties. First, with some typical tricks for solving functional equations, the substitution method is often useful. It consists in the fact that variables are replaced by some new functions (perhaps constants), which allow you bring the equation to a more convenient form.

Composition of functions

You, of course, noticed that the number of initial, basic functions studied in the school mathematics course are linear, power, trigonometric functions. Other functions are derived from the basic ones using compositions and algebraic actions.

Thus, the function $f(x)=sin(2x+1)$ is *a* composition of the linear function $g(x)=2x+1$ and the trigonometric function $h(x)=\sin x$, i. e.

$f(x)= h(g(x))=(L\text{og})(x)$.

The function $f(x)=lgarcsinx$ is obtained as *a* result of the composition of the functions $g(x)= \arcsin x$ and $h(x)=\lg x$.

Note that in the composition definition (Log) contains those values of *x* from *D*(*g*), for which $g(x)\in D(h)$. In the last example, *D*(*g*)=[-1;1], *D*(*0;∞*). Since *arcsinx*>0 for $x\in(0;1]$, then *D*(*f*)= (0;1]. If we take the composition of the same functions in reverse order, that is, the function *f*(*x*)=*arcsinlgx*, then we get $f(x)=\left[\frac{1}{10};10\right]$.

Solve the following problem

Task 1. Find all the functions of *y*= *f*(*x*) such that

$$f(x)+2f\left(\frac{1}{\alpha}\right)=3x \qquad (x\neq 0) \qquad (1)$$

Solution. Suppose there is a function *f*(*x*) that satisfies the given equation. Replacing x with $\frac{1}{\mathrm{x}}$, we get

$$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x} \qquad (2) \qquad \text{and} \qquad f(x)+2f\left(\frac{1}{x}\right)=3x.$$

We obtain the system of equations (1) and (2).

From here

$$f(x)=\frac{2}{x}-x.$$

Task 2. Find all the functions of *y*= *f*(*x*) such that

$$2f(1-x)+1=k\cdot f(x) \qquad (3)$$

Solution. Suppose there is a function *f*(*x*) that satisfies the given equation. Replacing *x* with, $1-x$ we get

$$2f(x)+1=(1-x)\cdot f(1-x) \qquad (4)$$

From (3) we find $f(1-x)=\frac{1}{2}(xf(x)-1)$.

Substituting $f(1-x)$ and equation (4), we get

$$2f(x)+1=(1-x)\cdot\frac{1}{2}\cdot(xf(x)-1), \text{ from where } f(x)=\frac{x-3}{x^2-x+4}.$$

By direct verification, we make sure that the resulting function satisfies equation (3).

In the considered equation under the sign of the unknown function $f_1=x$ and $f_2=1-x$. Replacing *x* with, $1-x$ transforms the functions f_1 and f_2 into each other. Substitution of $x\rightarrow 1-x$ yields another equation containing $f(x)$ and $f(1-x)$. We reduced the solution of a system of a system of two linear equations with two unknowns.

Consider now a more complex problem.

Task 3. Solve equation

$$xf(x)+2f\left(\frac{x-1}{x+1}\right)=1 \qquad (5)$$

Solution. Let's try to act in the same way as in the second task. Change $x \to \frac{x-1}{x+1}$. We get

$$\frac{x-1}{x+1}f\left(\frac{x-1}{x+1}\right)+2f\left(-\frac{1}{x}\right)=1 \qquad (6)$$

Along with the expressions $f(x)$ and $f\left(\frac{x-1}{x+1}\right)$, we have a new "unknown" $-f\left(-\frac{1}{x}\right)$.

Let's try to apply one more substitution to (5): $x \to --\frac{1}{x}$. We have

$$-\frac{1}{x}f\left(-\frac{1}{x}\right)+2f\left(\frac{x+1}{1-x}\right)=1 \qquad (7)$$

In addition $f\left(-\frac{1}{x}\right)$ an "unwanted" expression $f\left(\frac{x+1}{1-x}\right)$ in (7). And finally, luck.

We get the equation

$$\frac{x+1}{1-x}f\left(\frac{x+1}{1-x}\right)+2f(x)=1 \qquad (8)$$

Where new unknowns did not arise- a system of four linear equations (5)-(8) with four unknowns $f(x)$, $f\left(\frac{x-1}{x+1}\right)$, $f\left(-\frac{1}{x}\right)$ and $f\left(\frac{x+1}{1-x}\right)$

or

$$\begin{cases} xf(x)+2f\left(\frac{x-1}{x+1}\right)=1 \\ \frac{x-1}{x+1}f\left(\frac{x-1}{x+1}\right)+2f\left(-\frac{1}{x}\right)=1 \\ -\frac{1}{x}f\left(-\frac{1}{x}\right)+2f\left(\frac{x+1}{1-x}\right)=1 \\ \frac{x+1}{1-x}f\left(\frac{x+1}{1-x}\right)+2f(x)=1 \end{cases}$$

Eliminating $f\left(\frac{x-1}{x+1}\right)$, $f\left(-\frac{1}{x}\right)$ $f\left(\frac{x+1}{1-x}\right)$ in succession, we find

$$f(x)=\frac{4x^2-x+1}{5x(x-1)} \qquad (x \neq -1, x \neq 0, x \neq 1).$$

When solving functional equations, the method from particular to general is often used. Let's look at some examples. Does there exist a function f such that for any real x and y the equality

$$f(x)+f(y)=xy$$

Solution. Using the fact that *x* and *y* are any numbers, we put in the equality

$y=x,\ 2f(x)=x^2$, i.e. $f(x)=\dfrac{x^2}{2}$. Let us make a check, which is mandatory in this case (if for $y=x$, the function $f(x)=\dfrac{x^2}{2}$, is obtained, which, when substituted into a functional equation, turns It into an identity, then this does not mean that a similar situation will occur with $y\neq x$.

We get

$\left(\dfrac{x^2}{2}\right)+\left(\dfrac{y^2}{2}\right)=xy$, but this equality is not an identity. Therefore, such a function does not exist.

Task 4. Do there exist functions *f* and *g* such that for any real x and y the equality

$$f(x)\cdot g(x)=x+y+1$$

Solution. Let us assume that such functions *f* and *g* exist, ad we will try to find them. We put in the functional equation $x=y=0$, then

$$f(0)\cdot g(0)=1$$

Now we put $x=0$ in it, then

$$f(0)\cdot g(y)=y+1$$

Finally, in the original equality we put $y=0$:

$$f(x)\cdot g(0)=x+1.$$

Multiply the last two equalities:

$$f(0)\cdot g(y)\cdot f(x)\cdot g(0)=(y+1)(x+1)$$

But in this equality $f(0)\cdot g(0)=1$, therefore,

$$f(x)\cdot g(y)=(x+1)(y+1)$$

Let's check by substituting it into the original equality:

$(x+1)(y+1)=x+y+1,$

Obviously, the last equality does not hold for all *x* and *y*. We came to a contradiction. Therefore, there are no such *x* function *f* and *g*.

Task 5. Find all functions *f* satisfying the equation

$$f(x)+(x-2)f(1)+3f(0)=x^3+2,\qquad x\in R$$

Solution. Substitution in the equation x=1 with, x=0

$$\begin{cases} f(1)+(x-2)f(1)+3f(0)=1^3+2 \\ f(0)+(0-2)f(1)+3f(0)=0^3+2 \end{cases} \Leftrightarrow \begin{cases} f(0)=1 \\ f(1)=1 \end{cases}$$

Whence we get

$$f(x)=x^3+2-f(1)\cdot(x-2)-3f(0)=x^3-x+1 \ .$$

By checking, we make sure that the found function satisfies the equation.

ans: $f(x)=x^3-x+1$.

It is possible to outline a general method for solving some functional equations using the concept of a group of functions, but we restrict ourselves to the method of substitutions.

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